B.Tech-3rd(All Br.)
Mathematics-III

Full Marks : 70

Time : 3 hours

Answer **six** questions including **Q. No. 1** which is compulsory

The figures in the right-hand margin indicate marks

1. Answer the following questions : 2×10

   (*a*) Express $\sin(1 + i)$ in $a + ib$ form.

   (*b*) Find all values of $(-1 + i)^{-3i}$.

   (*c*) Show that the function $\bar{z}$ is nowhere differentiable.

   (*d*) Find $\oint_C \bar{z}\, dz$ where C is the unit circle about the origin.

   (*e*) Write the maximum modulus theorem.

(*f*) Obtain the partial differential equation from $z = axy + b$ by eliminating arbitrary constants a and b.

(*g*) Classify the one dimensional wave equation as parabolic or hyperbolic.

(*h*) Evaluate $\int_C y^2\,dx - 2x^2\,dy$ where C is the parabola $y = x^2$ from (0, 0) to (2, 4).

(*i*) Find a unit normal vector of the surface represented by $x^2 + y^2 + z^2 = 36$.

(*j*) Evaluate $\int_C \frac{dz}{z-1}$ $C : |z| = 1$.

2. (*a*) Find all solutions of the equation $z^4 - (3 + 6i)\,z^2 - 8 + 6i = 0$. 5

(*b*) Determine the set of all z satisfying the equation $|z| + Re(z) = 0$. 5

3. (*a*) Find the linear fractional transformation $w = f(z)$ which maps $i, 1, 2$ in z-domain to $i, -i, 1 + i$ in w-domain respectively. 5

(*Continued*)

(b) Determine the radius of convergence and open disk of convergence of the power series $\sum_{n=0}^{\infty} \frac{i^n}{2^{n+1}}(z+8i)^n$. 5

4. (a) Evaluate $\oint_C \frac{z^4}{z-2i}\,dz$ where C is any closed path enclosing $2i$. 5

(b) Evaluate $\oint_C \frac{z\sin(3z)}{(z+4)^3}\,dz$, where C is the circle $|z-2i|=9$. 5

5. (a) Evaluate $\oint_C \frac{z}{\sinh^2(z)}\,dz$, where C is the circle of radius 1 about $\frac{1}{2}$. 5

(b) Suppose f is differentiable in an open disk about zero and satisfies $f''(z) = 2f(z) + 1$. Suppose $f(0) = 1$ and $f'(0) = i$. Find the Maclaurin expansion of $f(z)$. 5

6. Find the temperature at any arbitrary time in a bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$. 10

7. Find the deflection of a string stretched between two fixed points at a distance $2c$ apart when the string is initially at rest and have initial velocity $v(x)$, where

$$v(x) = \begin{cases} \frac{x}{c}, & 0 < x < c \\ \frac{2c - x}{c}, & c < x < 2c \end{cases}$$ 10

8. (*a*) Using residues evaluate $\int_0^{2\pi} \frac{d\theta}{3 + \sin\theta}$. 5

(*b*) Evaluate $\int_{-\infty}^{\infty} \frac{dn}{(n^2 + 1)^2}$. 5